Fuzzy Space Time, Quantum Geometry and Cosmology

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Abstract

It is argued that a noncommutative geometry of spacetime leads to a reconciliation of electromagnetism and gravitation while providing an underpinning to Weyl's geometry. It also leads to a cosmology consistent with observation. A few other ramifications are also examined.

1 Introduction

The Theory of Relativity (Special and General) and Quantum Theory have been often described as the two pillars of twentieth century physics. Yet it was almost as if Rudyard Kipling's "The twain shall never meet" was true for these two intellectual achievements. For decades there have been fruitless attempts to unify electromagnetism and gravitation, or Quantum Theory and General Relativity. As Wheeler put it [1], the problem has been, how to incorporate curvature into Quantum Theory or spin half into General Relativity. At the same time it is also remarkable that both these disparate theories share one common platform: An underlying differentiable space time manifold, be it the Reimannian spacetime of General Relativity or the Minkowski spacetime of Relativistic Quantum Theory (including Quantum Field Theory). However this underlying common feature has been questioned by Quantum Gravity on the one hand and Quantum SuperStrings on the other, which try to unify these two branches (Cf.ref.[2] and several references therein). We will now argue that unification and a geometrical structure for Quantum Theory are possible if differentiable spacetime is discarded in favour of fuzzy spacetime. At the same time this has many ramifications and leads to a cosmology which is consistent with the latest iconoclastic observations, for example that the universe is accelerating and expanding for ever while the fine structure constant seems to be changing with time.

2 Quantum Geometry

One of the earliest attempts to unify electromagnetism and gravitation, was Weyl's gauge invariant geometry. The basic idea was [3] that while

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

was invariant under arbitrary transformations in General Relativity, a further invariant, namely,

$$\Phi_\mu dx^\mu \tag{2}$$

which is a linear form should be introduced. $g_{\mu\nu}$ in (1) would represent the gravitational potential, and Φ_μ of (2) would represent the electromagnetic field potential. As Weyl observed, "The world is a $3+1$ dimensional metrical manifold; all physical field - phenomena are expressions of the metrics of the world. (Whereas the old view was that the four-dimensional metrical continuum is the scene of physical phenomena; the physical essentialities themselves are, however, things that exist "in" this world, and we must accept them in type and number in the form in which experience gives us cognition of them: nothing further is to be "comprehended" of them.)$\cdots$"
This was a bold step, because it implied the relativity of magnitude multiplied effectively on all components of the metric tensor $g_{\mu\nu}$ by an arbitrary function of the coordinates. However, the unification was illusive because the $g_{\mu\nu}$ and Φ_μ were really independent elements[4].
A more modern treatment is recapitulated below [5].
The above arbitrary multiplying factor is normalised and we require that,

$$|g_{\mu\nu}| = -1, \tag{3}$$

For the invariance of (3), $g_{\mu\nu}$ transforms now as a tensor density of weight minus half, rather than as a tensor in the usual theory. The covariant derivative now needs to be redefined as

$$T^{\iota\cdots}_{\kappa\cdots,\sigma} = T^{\iota\cdots}_{\kappa\cdots,\sigma} + \Gamma^\iota_{\rho\sigma} T^{\rho\cdots}_{\kappa\cdots} - \Gamma^\rho_{\kappa\sigma} T^{\iota\cdots}_{\rho\cdots} - n T^{\iota\cdots}_{\kappa\cdots} \Phi_\sigma, \tag{4}$$

In (4) we have introduced the Φ_μ, and n is the weight of the tensor density. This finally leads to (Cf.ref.[5] for details).

$$\Phi_\sigma = \Gamma^\rho_{\rho\sigma}, \tag{5}$$

Φ_μ in (5) is identified with the electromagnetic potential, while $g_{\mu\nu}$ gives the gravitational potential as in the usual theory. The affine connection is now given by

$$\Gamma^\lambda_{\iota\kappa} = \frac{1}{2}g^{\lambda\sigma}(g_{\iota\sigma,\kappa} + g_{\kappa\sigma,\iota} - g_{\iota\kappa,\sigma}) + \frac{1}{4}g^{\lambda\sigma}(g_{\iota\sigma}\Phi_\kappa + g_{\kappa\sigma}\Phi_\iota - g_{\iota\kappa}\Phi_\sigma) \equiv \left(\begin{array}{c} \lambda \\ \iota\kappa \end{array} \right) \tag{6}$$

The essential point, and this was the original criticism of Einstein and others, is that in (6), $g_{\mu\nu}$ and Φ_μ are independent entities.
Let us now analyze the above from a different perspective. Let us write the product $dx^\mu dx^\nu$ of (1) as a sum of half its anti-symmetric part and half the symmetric part. The invariant line element in (1) now becomes $(h_{\mu\nu} + \hbar_{\mu\nu})dx^\mu dx^\nu$ where h and $\hbar$ denote the anti-symmetric and symmetric parts respectively of g. h would vanish unless the commutator

$$[dx^\mu, dx^\nu] \approx l^2 \neq 0 \tag{7}$$

l being some fundamental minimum length. In this case, under reflection, $h_{\mu\nu} \to -h_{\mu\nu}$ as in the case of the tensor density metric tensor above.
To proceed further, we observe that the noncommutative geometry given in (7) was studied by Snyder and others. In this case it has been shown in detail by the author [6, 7] that under an infinitesimal Lorentz transformation of the wave function,

$$|\psi' >= U(R)|\psi > \tag{8}$$

we get

$$\psi'(x_j) = [1 + \imath\epsilon(\imath\epsilon l_{jk}x_k\frac{\partial}{\partial x_j}) + 0(\epsilon^2)]\psi(x_j) \tag{9}$$

Equation (9) has been shown to lead to the Dirac equation when l is the Compton wavelength. Indeed, Dirac himself had noted that his electron equation needed an average over spacetime intervals of the order of the Compton scale to remove zitterbewegung effects and give meaningful physics. This again has recently been shown to be symptomatic of an underlying fuzzy spacetime described by a noncommutative space time geometry (7) [8].

The point here is that under equation (7), the coordinates $x^\mu \to \gamma^{(\mu)} x^{(\mu)}$ where the brackets with the superscript denote the fact that there is no summation over the indices. Infact, in the theory of the Dirac equation it is well known [9].that,

$$\gamma^k \gamma^l + \gamma^l \gamma^k = -2g^{kl} I \tag{10}$$

where γ's satisfy the usual Clifford algebra of the Dirac matrices, and can be represented by

$$\gamma^k = \sqrt{2} \begin{pmatrix} 0 & \sigma^k \\ \sigma^{k*} & 0 \end{pmatrix} \tag{11}$$

where σ's are the Pauli matrices. As noted by Bade and Jehle (Cf.ref.[9]), we could take the σ's or γ's in (10) and (11) as the components of a contravariant world vector, or equivalently one could take them to be fixed matrices, and to maintain covariance, to attribute new transformation properties to the wave function, which now becomes a spinor (or bi-spinor). This latter has been the traditional route, because of which the Dirac wave function has its bi-spinorial character. In this latter case, the coordinates retain their usual commutative character. It is only when we consider the equivalent former alternative, that we return to the noncommutative geometry (7).
That is in the usual commutative spacetime the Dirac spinorial wave functions conceal the noncommutative character (7).
Indeed we can verify all these considerations in a simple way as follows:
First let us consider the usual space time. This time the Dirac wave function is given by

$$\psi = \begin{pmatrix} \chi \\ \Theta \end{pmatrix},$$

where χ and Θ are spinors. It is well known that under reflection while the so called positive energy spinor Θ behaves normally, $\chi \to -\chi, \chi$ being the so called negative energy spinor which comes into play at the Compton scale [10]. Because of this property as shown in detail [7], there is now a covariant derivative given by, in units, $\hbar = c = 1$,

$$\frac{\partial \chi}{\partial x^\mu} \to [\frac{\partial}{\partial x^\mu} - nA^\mu]\chi \tag{12}$$

where

$$A^\mu = \Gamma^{\mu\sigma}_\sigma = \frac{\partial}{\partial x^\mu} log(\sqrt{|g|}) \tag{13}$$

Γ denoting the Christofell symbols.
A^μ in (13)is now identified with the electromagnetic potential, exactly as in Weyl's theory.
What all this means is that the so called ad hoc feature in Weyl's unification theory is really symptomatic of the underlying noncommutative space time geometry (7). Given (7) we get both gravitation and electromagnetism in a unified picture.
Let us now consider the above ideas in the context of the deBroglie-Bohm formulation [11]. We start with the Schrodinger equation

$$\imath\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{14}$$

In (14), the substitution

$$\psi = Re^{\imath S/\hbar} \tag{15}$$

where R and S are real functions of $\vec{r}$ and t, leads to,

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \tag{16}$$

$$\frac{1}{\hbar}\frac{\partial S}{\partial t} + \frac{1}{2m}(\vec{\nabla}S)^2 + \frac{V}{\hbar^2} - \frac{1}{2m}\frac{\nabla^2 R}{R} = 0 \tag{17}$$

where

$$\rho = R^2, \vec{v} = \frac{\hbar}{m}\vec{\nabla}S$$

and

$$Q \equiv -\frac{\hbar^2}{2m}(\nabla^2 R/R) \tag{18}$$

Using the theory of fluid flow, it is well known that (16) and (17) lead to the Bohm alternative formulation of Quantum Mechanics. In this theory there is a hidden variable namely the definite value of position while the so called Bohm potential Q can be non local, two features which do not find favour with physicists.
It must be noted that in Weyl's geometry, even in a Euclidean space there is a covariant derivative and a non vanishing curvature R.
Santamato [12, 13, 14] exploits this latter fact, within the context of the deBroglie-Bohm theory and postulates a Lagrangian given by

$$L(q,\dot{q},t) = L_c(q,\dot{q},t) + \gamma(\hbar^2/m)R(q,t),$$

He then goes on to obtain the equations of motion like (14),(15), etc. by invoking an Averaged Least Action Principle

$$I(t_0, t_1) = E\left\{\int_{t_0}^{t} L^*(q(t,\omega), \dot{q}(t,\omega), t)dt\right\}$$
$$= \text{minimum}, \quad (19)$$

with respect to the class of all Weyl geometries of space with fixed metric tensor. This now leads to the Hamilton-Jacobi equation

$$\partial_t S + H_c(q, \nabla S, t) - \gamma(\hbar^2/m)R = 0, \quad (20)$$

and thence to the Schrodinger equation (in curvi-linear coordinates)

$$\imath\hbar\partial_t\psi = (1/2m)\left\{[(\imath\hbar/\sqrt{g})\partial_\imath\sqrt{g}A_\imath]g^{\imath k}(\imath\hbar\partial_k + A_k)\right\}\psi$$
$$+[V - \gamma(\hbar^2/m)\dot{R}]\psi = 0, \quad (21)$$

As can be seen from the above, the Quantum potential Q is now given in terms of the scalar curvature R.
We have already related the arbitrary functions Φ of Weyl's formulation with a noncommutative spacetime geometry (7).
This legitimises Santamato's postulative approach of extending the deBroglie-Bohm formulation.
At an even more fundamental level, this formalism gives us the rationale for the deBroglie wave length itself. Because of the noncommutative geometry in (7) space becomes multiply connected, in the sense that a closed circuit cannot be shrunk to a point within the interval. Let us consider the simplest case of double connectivity. In this case, if the interval is of length λ, we will have,

$$\Gamma \equiv \int_c m\vec{V}\cdot d\vec{r} = h\int_c \vec{\nabla}S\cdot d\vec{r} = h\oint dS = mV\pi\lambda = \pi h \quad (22)$$

whence

$$\lambda = \frac{h}{mV} \quad (23)$$

In (22), the circuit integral was over a circle of diameter λ. Equation (23) shows the emergence of the deBroglie wavelength. This follows from the noncommutative geometry of space time, rather than the physical Heisenberg Uncertainty Principle. Remembering that Γ in (22) stands for the angular momentum, this is also the origin of the Wilson-Sommerfeld quantization rule, an otherwise mysterious Quantum Mechanical prescription.
What we have done is to develop a Quantum Geometry, based on (7).

3 Cosmology

In recent years the work of Perlmutter and co-workers has shown that the universe is not only not descelerating, it is actually accelerating, and would continue to expand for ever. The work of Webb and co-workers on the other hand brings out an equally iconoclastic observation: The hallowed fine structure constant is actually slowly decreasing with time. Suddenly dark matter has been discarded in favour of dark energy and a cosmological constant.
We first observe that the concept of a Zero Point Field (ZPF) or Quantum Vacuum (or Ether) is an idea whose origin can be traced back to Max Planck himself. Quantum Field Theory attributes the ZPF to the virtual Quantum Effects of an already present electromagnetic field. There is another approach, sometimes called Stochastic Electrodynamics which treats the ZPF as primary and attributes to it Quantum Mechanical effects [15, 16]. It may be observed that the ZPF results in the well known experimentally verified Casimir effect [17, 18]. We would also like to point out that contrary to popular belief, the concept of Ether has survived over the decades through the works of Dirac, Vigier, Prigogine, String Theorists like Wilzeck and others [19]-[27]. It appears that even Einstein himself continued to believe in this concept [28].
We would first like to observe that the energy of the fluctuations in the background electromagnetic field could lead to the formation of elementary particles. Infact it is known that this energy of fluctuation in a region of length l is given by [1]

$$B^2 \sim \frac{\hbar c}{l^4}$$

In the above if l is taken to be the Compton wavelength of a typical elementary particle, then we recover its energy mc^2, as can be easily verified. It may be mentioned that Einstein himself had believed that the electron was a result of such condensation from the background electromagnetic field (Cf.[11] for details). Infact this formation of particles could be likened to the formation of Benard cells in a phase transition [29], as we will see briefly in the next section. We also take the pion to represent a typical elementary particle, as in the literature.
To proceed, as there are $N \sim 10^{80}$ such particles in the universe, we get

$$Nm = M \tag{24}$$

where M is the mass of the universe.
In the following we will use N as the sole cosmological parameter.
Equating the gravitational potential energy of the pion in a three dimensional isotropic sphere of pions of radius R, the radius of the universe, with the rest energy of the pion, we can deduce the well known relation [11]

$$R \approx \frac{GM}{c^2} \tag{25}$$

where M can be obtained from (24).
We now use the fact that given N particles, the fluctuation in the particle number is of the order $\sqrt{N}$[30], while a typical time interval for the fluctuations is $\sim \hbar/mc^2$, the Compton time. We will come back to this point later. So we have

$$\frac{dN}{dt} = \frac{\sqrt{N}}{\tau}$$

whence on integration we get,

$$T = \frac{\hbar}{mc^2}\sqrt{N} \tag{26}$$

We can easily verify that equation (26) is indeed satisfied where T is the age of the universe. Next by differentiating (25) with respect to t we get

$$\frac{dR}{dt} \approx HR \tag{27}$$

where H in (27) can be identified with the Hubble Constant, and using (25) is given by,

$$H = \frac{Gm^3c}{\hbar^2} \tag{28}$$

Equation (24), (25) and (26) show that in this formulation, the correct mass, radius and age of the universe can be deduced given N as the sole cosmological or large scale parameter. Equation (28) can be written as

$$m \approx \left(\frac{H\hbar^2}{Gc}\right)^{\frac{1}{3}} \tag{29}$$

Equation (29) has been empirically known as an "accidental" or "mysterious" relation. As observed by Weinberg[31], this is unexplained: it relates a single

cosmological parameter H to constants from microphysics. We will touch upon this micro-macro nexus again. In our formulation, equation (29) is no longer a mysterious coincidence but rather a consequence.
As (28) and (27) are not exact equations but rather, order of magnitude relations, it follows that a small cosmological constant $\wedge$ is allowed such that

$$\wedge \leq 0(H^2)$$

This is consistent with observation and shows that $\wedge$ is very very small - this has been a puzzle, the so called cosmological constant problem [32]. But it is explained here.
To proceed we observe that because of the fluctuation of $\sim \sqrt{N}$ (due to the ZPF), there is an excess electrical potential energy of the electron, which infact we have identified as its inertial energy. That is [30],

$$\sqrt{N}e^2/R \approx mc^2.$$

On using (25) in the above, we recover the well known Gravitation-electromagnetism ratio viz.,

$$e^2/Gm^2 \sim \sqrt{N} \approx 10^{40} \tag{30}$$

or without using (25), we get, instead, the well known so called Eddington formula,

$$R = \sqrt{N}l \tag{31}$$

Infact (31) is the spatial counterpart of (26). If we combine (31) and (25), we get,

$$\frac{Gm}{lc^2} = \frac{1}{\sqrt{N}} \propto T^{-1} \tag{32}$$

where in (32), we have used (26). Following Dirac (cf.also [33]) we treat G as the variable, rather than the quantities $m, l, c$and$\hbar$ (which we will call micro physical constants) because of their central role in atomic (and sub atomic) physics.
Next if we use G from (32) in (28), we can see that

$$H = \frac{c}{l} \quad \frac{1}{\sqrt{N}} \tag{33}$$

Thus apart from the fact that H has the same inverse time dependance on T as G, (33) shows that given the microphysical constants, and N, we can

deduce the Hubble Constant also, as from (33) or (28).
Using (24) and (25), we can now deduce that

$$\rho \approx \frac{m}{l^3} \quad \frac{1}{\sqrt{N}} \tag{34}$$

Next (31) and (26) give,

$$R = cT \tag{35}$$

(34) and (35) are consistent with observation.
The above model predicts an ever expanding and possibly accelerating universe whose density keeps decreasing. This seemed to go against the accepted idea that the density of the universe equalled the critical density required for closure.
The above cosmology exhibits a time variation of the gravitational constant of the form

$$G = \frac{\beta}{T} \tag{36}$$

Indeed this is true in a few other schemes also, including Dirac's cosmology (Cf. [34]). Interestingly it can be shown that such a time variation can explain the precession of the perihelion of Mercury (Cf.[35]). It can also provide an alternative explanation for dark matter and the bending of light while the Cosmic Microwave Background Radiation is also explained (Cf.[11]).
It is also possible to deduce the existence of gravitational waves given (36). To see this quickly let us consider the Poisson equation for the metric $g_{\mu\nu}$

$$\nabla^2 g_{\mu\nu} = G\rho u_\mu u_\nu \tag{37}$$

The solution of (37) is given by

$$g_{\mu\nu} = G \int \frac{\rho u_\mu u_\nu}{|\vec{r} - \vec{r}'|} d^3\vec{r} \tag{38}$$

Indeed equations similar to (37) and (38) hold for the Newtonian gravitational potential also. If we use the second time derivative of G from (36) in (38), along with (37), we can immediately obtain the D'alembertian wave equation for gravitational waves, instead of the Poisson equation:

$$Dg_{\mu\nu} \approx 0$$

where D is the D'alembertian.
Recently a small variation with time of the fine structure constant has been

detected and reconfirmed by Webb and coworkers [36, 37]. This observation is consistent with the above cosmology. We can see this as follows. We use an equation due to Kuhne [38]

$$\frac{\dot{\alpha}_z}{\alpha_z} = \alpha_z \frac{\dot{H}_z}{H_z}, \tag{39}$$

If we now use the fact that the cosmological constant Λ is given by

$$\Lambda \leq 0(H^2) \tag{40}$$

as can be seen from (27), in (39), we get using (40),

$$\frac{\dot{\alpha}_z}{\alpha_z} = \beta H_z \tag{41}$$

where $\beta < -\alpha_z < -10^{-2}$.
Equation (41) can be shown to be the same as

$$\frac{\dot{\alpha}_z}{\alpha_z} \approx -1 \times 10^{-5} H_z. \tag{42}$$

which is the same as Webb's result.
We give another derivation of (42) in the above context wherein, as the number of particles in the universe increases with time, we go from the Planck scale to the Compton scale.
This can be seen as follows: In equation (30), if the number of particles in the universe, $N = 1$, then the mass m would be the Planck mass. In this case the classical Schwarzschild radius of the Planck mass would equal its Quantum Mechanical Compton wavelength. To put it another way, all the energy would be gravitational (Cf.[11] for details). However as the number of particles N increases with time, according to (26), gravitation and electromagnetism get differentiated and we get (30) and the Compton scale.
It is known that the Compton length, due to zitterbewegung causes a correction to the electrostatic potential which an orbiting electron experiences, rather like the Darwin term [10].
Infact we have

$$\langle \delta V \rangle = \langle V(\vec{r} + \delta\vec{r}) \rangle - V\langle (\vec{r}) \rangle$$
$$= \langle \delta r \frac{\partial V}{\partial r} + \frac{1}{2} \sum_{\imath j} \delta r_\imath \delta r_j \frac{\partial^2 V}{\partial r_\imath \partial r_j} \rangle$$

$$\approx 0(1)\delta r^2 \nabla^2 V \quad (43)$$

Remembering that $V = e^2/r$ where $r \sim 10^{-8}cm$, from (43) it follows that if $\delta r \sim l$, the Compton wavelength then

$$\frac{\Delta\alpha}{\alpha} \sim 10^{-5} \quad (44)$$

where $\Delta\alpha$ is the change in the fine structure constant from the early universe. (44) is an equivalent form of (42) (Cf.ref.[38, 39]), and is the result originally obtained by Webb et al (Cf.refs.[39, 40]).

Infact there is another test for the variation of G.We would now like to show that this model also explains the observed decrease in the orbital period of the binary pulsar PSR $1913 + 16$, otherwise attributed to as yet undetected gravitational waves [40].

In general in schemes in which G the universal constant of gravitation decreases with time, it is to be expected that gradually the size of the orbit and the time period would increase, with an overall decrease in energy. In any case all this becomes more relevant in the light of the above latest observations about the fine structure constant.

But in the present case as we will show, the gravitational energy of the binary system, $\frac{GMm}{L}$ remains constant, where M is the mass of the central object and L the mean distance between the objects. This is because the decrease in G is compensated by an increase in the material content of the system, according to the above model.

In fact the energy lost is given by $\frac{GM}{TL}$ (per unit mass of the orbiting object - in any case the mass of the orbiting object does not feature in the dynamical equations). Further from what we saw $\frac{1}{\sqrt{N}\tau} = \frac{1}{T}$ particles appear from the Quantum Vacuum per second, per particle in the universe. So the energy gained in this process is $\frac{GM}{TL}$ per second. This follows, if we write $M = n \times m$, where n is the number of typical elementary particles in the central body and m their mass.

As can be seen from the above the energy lost per second is compensated by the energy gained and thus the total gravitational energy of the binary system remains constant.

Let us now consider an object revolving about another object, as in the case of the binary pulsar. The gravitational energy of the system is now given by,

$$\frac{GMm}{L} = const.$$

Whence

$$\frac{\mu}{L} \equiv \frac{GM}{L} = const. \qquad (45)$$

For variable G we have

$$\mu = \mu_0 - tK \qquad (46)$$

where

$$K \equiv \dot{\mu} \qquad (47)$$

We take $\dot{\mu}$ to be a constant, in view of the fact that G varies very slowly, as can be seen from (1).
To preserve (45), we should have

$$L = L_0(1 - \alpha K)$$

Whence on using (46)

$$\alpha = \frac{t}{\mu_0} \qquad (48)$$

We shall consider t, to be the period of revolution. Using (48) it follows that

$$\delta L = -\frac{LtK}{\mu_0} \qquad (49)$$

We also know from theory,

$$t = \frac{2\pi}{h} L^2 = \frac{2\pi}{\sqrt{\mu}} \qquad (50)$$

$$t^2 = \frac{4\pi^2 L^3}{\mu} \qquad (51)$$

Using (49), (50) and (51), a little manipulation gives

$$\delta t = -\frac{2t^2 K}{\mu_0} \qquad (52)$$

(49) and (52) show that there is a decrease in the size of the orbit, as also in the orbital period. Before proceeding further we note that such a decrease in the orbital period has been observed in the case of binary pulsars [40, 41]. Let us now apply the above considerations to the case of the binary pulsar PSR $1913 + 16$ observed by Taylor and co-workers (Cf.ref.[41]). In this case

it is known that, t is 8 hours while v, the orbital speed is $3 \times 10^7 cms$ per second. It is easy to calculate from the above

$$\mu_0 = 10^4 \times v^3 \sim 10^{26}$$

which gives $M \sim 10^{33} gms$, which of course agrees with observation. Further we get using (47)

$$\Delta t = \eta \times 10^{-5} sec/yr, \eta \leq 8 \tag{53}$$

Indeed (53) is in good agreement with the carefully observed value of $\eta \approx 7.5$ (Cf.refs.[40, 41]).
It may be remarked that this same effect has been interpreted as being due to gravitational radiation, even though there are some objections to the calculation in this case (Cf.ref.[41]).
We will now try to explain the Pioneer spacecrafts' large anomalous acceleration which has been studied for several years by Anderson and co-workers and has remained a long standing puzzle [42].
From the energy conservation in central orbits, viz.,

$$\frac{m}{2}(\dot{r}^2 + r^2\dot{\Theta}^2) - \frac{GM}{r} = const.,$$

we get, on differentiation, using the effect of the variable G (32), an extra inward acceleration,

$$a_r = \frac{GM}{t_0 r \dot{r}} \tag{54}$$

where t_0 is the age of the universe.
On the other hand, from the standard equation for the orbit

$$\frac{l}{r} = 1 + ecos\Theta,$$

where $l = \frac{(r^2\dot{\Theta})^2}{GM}$, we get, differentiating and using (32), the extra effect,

$$\dot{r} \approx \frac{r^{3/4}\nu}{t_0\sqrt{GM}}, \tag{55}$$

where $\nu = r\dot{\Theta}$.
Using (55) in (54) and the values for r and ν, viz., $\sim 10^{15}$ and $\sim 10^6$ respectively, we get

$$a_r \leq 10^{-6} cm/sec^2$$

This fits in very well with the results of Anderson et al that

$$a_r \sim 10^{-7} cm/sec^2$$

Thus, we can argue that the inexplicable large anomalous acceleration is a footprint of the variable G (32).

4 Critical Phenomena and Cosmology

It has been pointed out that in the universe at large, there appears to be the analogues of the Planck constant h at different scales [43, 44, 11] and several references therein. Infact we have

$$h_1 \sim 10^{93} \tag{56}$$

for super clusters;

$$h_2 \sim 10^{74} \tag{57}$$

for galaxies and

$$h_3 \sim 10^{54} \tag{58}$$

for stars. And

$$h_4 \sim 10^{34} \tag{59}$$

for Kuiper Belt objects. In equations (56) - (59), the $h_\imath$ play the role of the Planck constant, in a sense to be described below. The origin of these equations is related to the following empirical relations

$$R \approx l_1 \sqrt{N_1} \tag{60}$$

$$R \approx l_2 \sqrt{N_2} \tag{61}$$

$$l_2 \approx l_3 \sqrt{N_3} \tag{62}$$

$$R \sim l \sqrt{N} \tag{63}$$

and a similar relation for the KBO (Kuiper Belt objects)

$$L \sim l_4 \sqrt{N_4} \tag{64}$$

where $N_1 \sim 10^6$ is the number of superclusters in the universe, $l_1 \sim 10^{25} cms$ is a typical supercluster size $N_2 \sim 10^{11}$ is the number of galaxies in the universe and $l_2 \sim 10^{23} cms$ is the typical size of a galaxy, $l_3 \sim 1$ light years is a typical distance between stars and $N_3 \sim 10^{11}$ is the number of stars in a galaxy, R being the radius of the universe $\sim 10^{28} cms, N \sim 10^{80}$ is the number of elementary particles, typically pions in the universe and l is the pion Compton wavelength and $N_4 \sim 10^{10}, l_4 \sim 10^5 cm$, is the age of a typical KBO (with mass $10^{19} gm$ and L the width of the Kuiper Belt $\sim 10^{10} cm$ cf.ref.[11]).

The size of the universe is the size of a supercluster etc. from equations like (60)-(63), as described in the references turn up as the analogues of the Compton wavelength. For example we have

$$R = \frac{h_1}{Mc} \tag{65}$$

It has also been argued that these scaled Compton wavelengths and scaled Planck constants are the result of gravitational orbits described by

$$\frac{GM}{L} \sim v^2 \tag{66}$$

For example from (66) it follows that

$$MvL = h_2,$$

Whence we get (57).

While equations (60)-(63), resemble the Random Walk relations, this is not accidental.

Let us start with Nelson's well known equation of diffusion

$$\Delta k^2 = \nu \Delta t, \quad \nu = \frac{h}{m} \tag{67}$$

where ν is the diffusion constant, h the Planck constant and m the mass of a typical elementary particle like pion. We can see immediately from (67) that for velocities

$$\langle \frac{\Delta x}{\Delta t} \rangle = c$$

We recover the Compton wavelength (or more generally the deBroglie wavelength). Further it can be shown that from (67) we can recover the Random Walk equation (63) [45] define scaled diffusion constant to be $\nu_\imath$. Using (67) it immediately follows that

$$\Delta x_\imath^2 = \nu_\imath \Delta t_\imath \tag{68}$$

(68) is a diffusion equation at different scales. This in turn leads to equations (60) - (64), a similar one for the KBO, which were to start with empirical relations. The interesting point is that starting from (67) (or (68)), we can infact deduce the Schrodinger equation - either by using the Lagrangian, as is known, or even without invoking a Lagrangian (Cf.Appendix):

$$h_\imath \frac{\partial \psi}{\partial t} + \frac{h_\imath^2}{2m} \nabla^2 \psi = 0 \tag{69}$$

This provides a rationale for the scaled Compton lengths or scaled deBroglie lenghs referred to above.
Let us investigate the above considerations in a little more detail.
For this, we observe that the creation of particles from a Quantum vacuum (or pre space time) has been described in the references cited [46, 47]. It can be done within the context of the above Nelsonian Theory, in complete analogy with the creation of Benard cells at the critical point. The Nelsonian-Brownian process as described in (67) defines, first the Planck length, the shortest possible length and then the random process leads to the Compton scale (Cf.ref.[45]). This process is as noted (Cf.ref.[47]) a complete analogue of the phase transition associated with the Landau-Ginsburg equation [48]

$$-\frac{h^2}{2m} \nabla^2 \psi + \beta |\psi|^2 \psi = - \propto \psi \tag{70}$$

The parallel is not yet fully apparent, if we compare (70) and the Schrodinger equation (69). However this becomes clear if we consider how the Schrodinger equation itself can be deduced from the amplitudes of the Quantum vacuum, in which case we get

$$\imath \hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m'} \frac{\partial^2 \psi}{\partial x^2} + \int \psi^*(x') \psi(x) \psi(x') U(x') dx', \tag{71}$$

infact the correlation length from (70) is given by

$$\xi = (\frac{\gamma}{\propto})^{\frac{1}{2}}$$

which can be easily reduced to the Compton wavelength. In other words, the Schrodinger equation (69), via (71) describes the creation of particles, a la Benard cells in a Landau-Ginsburg like phase transition.
As is known, the interesting aspects of the critical point theory (Cf.ref.[48,

49]) are universality and scale. Broadly, this means that diverse physical phenomena follow the same route at the critical point, on the one hand, and on the other this can happen at different scales, as exemplified for example, by the course graining techniques of the renormalization group. To highlight this point we note that in critical point phenomena we have the reduced order parameter $\bar{Q}$ and the reduced correlation length bar $\bar{\xi}$. Near the critical point we have relations like (Cf.ref.[49])

$$(\bar{Q}) = |t|^{\beta}, (\bar{\xi}) = |t|^{-\nu}$$

Whence

$$\bar{Q}^{\nu} = \bar{\xi}^{\beta} \tag{72}$$

In (72) typically $\nu \approx 2\beta$. As $\sqrt{Q} \sim \frac{1}{\sqrt{N}}$ because $\sqrt{N}$ particles are created fluctuationally, given N particles, and in view of the fractal two dimensionality of the path

$$\bar{Q} \sim \frac{1}{\sqrt{N}}, \bar{\xi} = (l/R)^2$$

This gives

$$R = \sqrt{N}l$$

which is nothing but (63).

In other words the scaled Planck effects and the scaled Random Walk effects as typified by equations like (56)-(64) are the result of a critical point phase transition and subsequent course graining.

5 Discussion

1.In some ways the General Relativistic gravitational field resembles the electromagnetic field, particularly in certain approximations, as for example when the field is stationary or nearly so and the velocities are small. In this case the equations of General Relativity can be put into a form resembling those of Maxwell's Theory, and then the fields have been called Gravitoelectric and Gravitomagnetic [50]. Experiments have also been suggested for measuring the Gravitomagnetic force components for the earth [51].

We can ask whether such a consideration can be applied to elementary particles, if in fact they can be considered in the context of General Relativity. It may be mentioned that apart from Quantum Gravity, there have been three different approaches for studying elementary particles via General Relativity

[52, 53, 11] and references therein. We will now show that it is possible to extend the Gravitomagnetic and Gravitoelectric formulations to elementary particles within the framework of the theory developed in [11].
In [11], the linearized General Relativistic equations are seen to describe the properties of elementary particles, such as spin, mass, charge and even the very Quantum Mechanical anomalous gyromagnetic ratio $g = 2$, apart from several other characteristics [54, 55, 56, 57].
We merely report that the linearized equations of General Relativity, viz.,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{\mu\nu} = \int \frac{4T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' \tag{73}$$

where as usual,

$$T^{\mu\nu} = \rho u^u u^v \tag{74}$$

lead to the mass, spin, gravitational potential and charge of an electron, if we work at the Compton scale (Cf.ref.[11] for details). Let us now apply the macro Gravitoelectic and Gravitomagnetic equations to the above case. Infact these equations are (Cf.ref.[50]).

$$\nabla \cdot \vec{E}_g \approx -4\pi\rho, \nabla \times \vec{E}_g \approx -\partial \vec{H}_g / \partial t, etc. \tag{75}$$

$$\vec{E}_g = -\nabla\phi - \partial\vec{A}/\partial t, \quad \vec{H}_g = \nabla \times \vec{A} \tag{76}$$

$$\phi \approx -\frac{1}{2}(g_{00} + 1), \vec{A}_\imath \approx g_{0\imath}, \tag{77}$$

The subscripts g in the equations (75), (76), (77) are to indicate that the fields E and H in the macro case do not really represent the Electromagnetic field, but rather resemble them. Let us apply equation (76) to equation (73), keeping in mind equation (77). We then get, considering only the order of magnitude, which is what interests us here, after some manipulation

$$|\vec{H}| \approx \int \frac{\rho V}{r^2} \bar{r} \approx \frac{mV}{r^2} \tag{78}$$

and

$$|\vec{E}| = \frac{mV^2}{r^2} \tag{79}$$

V being the speed.
In (78) and (79) the distance r is much greater than a typical Compton

wavelength, to make the approximations considered in deriving the Gravitomagnetic and Gravitoelectric equations meaningful.
Remembering that we have

$$mVr \approx h,$$

the electric and magnetic fields in (78) and (79) now become

$$|\vec{H}| \sim \frac{h}{r^3}, |\vec{E}| \sim \frac{hV}{r^3} \qquad (80)$$

We now observe that (80) does not really contain the mass of the elementary particle. Could we get a further insight into this new force?
Indeed in the above linearized General Relativistic characterisation of the electron, it turns out that the electron can be represented by the Kerr-Newman metric (Cf.[11] for details). This incidentally also gives the anomalous gyromatgnetic ratio $g = 2$. This result has recently been reconfirmed by Nottale [58] from a totally different point of view, using scaled relativity. It is well known that the Kerr-Newman field has extra electric and magnetic terms (Cf.[59]), both of the order $\frac{1}{r^3}$, exactly as indicated in (80).
It may be asked if there is any candidate as yet for the above short range force. There is already one such candidate - the inexplicable $B_{(3)}$ [60] force mediated by massive photons and of short range, first detected in 1992 at Cornell and since confirmed by subsequent experiments. (It differs from the usual $B_{(1)}$ and $B_{(2)}$ fields of special relativity, mediated as they are, by massless photons.)
Interestingly, if we work with a massive vector field we can recover (79) and (80)[61]. In this case there is an upper limit on the mass of the photon $\sim 10^{-48}g$.
A Final Comment: It is quite remarkable that equations like (75), (76) and (77) which resemble the equations of electromagnetism, have in the usual macro considerations no connection whatsoever with electromagnetism except in appearance. This would seem to be a rather miraculous coincidence. In fact the above considerations of section 2 and linearized General Relativistic theory of the electron as also the Kerr-Newman metric formulation, demonstrate that the resemblence to electromagnetism is not an accident, because in this latter formulation, both electromagnetism and gravitation arise from the metric (Cf.also refs.[62, 54, 55, 11]).
2. Newman deduced the now famous Kerr-Newman metric alluded to nearly 40 years ago [63, 64]. There were two inexplicable features. The first was,

the use of complex coordinates, and why such coordinates somehow represent spin.
The second puzzling feature was, why should this General Relativistic metric so closely describe the anomalous gyro magnetic ratio, $g = 2$ [65, 1].
The difficulty is best brought out by the fact that the Kerr-Newman metric when applied to the electron throws up a naked singularity, when the distances become complex which is a reversal of the above situation, where complex coordinates were introduced in the first place.
We will now first see, why complex coordinates arise at all, and what the ramifications are: In the process we will get the answers to the above two puzzling questions.
To elaborate the above considerations, the horizon of the Kerr-Newman metric becomes, for the electron complex:

$$r = \frac{GM}{c^2} + \imath b, b \equiv \left(\frac{G^2Q^2}{c^8} + a^2 - \frac{G^2M^2}{c^4}\right)^{1/2} \tag{81}$$

However it should be noticed that the coordinates for a Dirac particle is given by

$$x = (c^2p_1H^{-1}t) + \frac{\imath}{2}c\hbar(\alpha_1 - cp_1H^{-1})H^{-1} \tag{82}$$

Interestingly the imaginary terms in both (81) and (82) are of the same order, namely the Compton wavelength, $\frac{\hbar}{mc}$ of the electron. For the complex or non-Hermitian coordinate in (82) Dirac had argued [66, 67] that our measurements of space time intervals are imprecise and infact averaged over the order of the Compton scale, and once such averages are taken, the imaginary or zitterbewegung term disappears, and we return to real or Hermitian coordinates.
Infact in Dirac's theory the operator $d_x \equiv \frac{d}{dx}$ is a purely imaginary operator, and is given by

$$\delta x(d_x + \bar{d}_x) = \delta x^2 d_x \bar{d}_x = 0$$

if

$$0(\delta x^2) = 0$$

as is tacitly assumed. However if

$$0(\delta x^2) \neq 0 \tag{83}$$

then the operator d_x becomes complex, and therefore, also the momentum operator, $p_x \equiv \imath\hbar d_x$ and the position operator. In other words if (83) holds

good then we have to deal with complex or non-Hermitian coordinates. The implication of this is that (Cf.[54] for details) space time becomes non- commutative as we saw in (7).
We also saw that this leads directly to the Dirac equation at the Compton scale.
In any case here is the mysterious origin of the complex coordinates and spin. The complex coordinates lead to the Kerr-Newman metric and the electron's field including the anomalous gyro magnetic ratio which are symptomatic of the electron's spin. It also means that the naked singularity is shielded by the fuzzy spacetime (Dirac's original averages over the zitterbewegung interval or equivalently the noncommutative geometry (7).
i) It may be noticed that while the original General Relativistic and Kerr-Newman formulations were for the macro universe, we on the other hand have applied it to the micro world.
ii). Once complex coordinates are introduced, as noted by Newman there is a change of character [65]: "Notice that the magnetic moment $\mu = ea$ can be thought of as the imaginary part of the charge times the displacement of the charge into the complex region.... We can think of the source as having a complex center of charge and that the magnetic moment is the moment of charge about the center of charge....In other words the total complex angular momentum vanishes around any point z^a on the complex world-line. From this complex point of view the spin angular momentum is identical to orbital, arising from an imaginary shift of origin rather than a real one... If one again considers the particle to be "localized" in the sense that the complex center of charge coincides with the complex center of mass, one again obtains the Dirac gyromagnetic ratio..."
Infact this above complexification of coordinates has also been worked out by Kaiser [68, 69], but for the Dirac electron. Kaiser found that such a complexification eliminates the unphysical zitterbewegung. Dirac himself had noted that zitterbewegung was a manifestation of the fact that while in a physical sense our space time intervals cannot be made arbitrarily small, in theory we consider point intervals, so that as noted above, once an averaging over the Compton scale is performed, zitterbewegung disappears.
What we would like to stress here is that, the complexification carried out by Newman or Kaiser were mathematical devices - the physical motivation or ramification was unclear. We argue that complexification is symptomatic of the fuzzy nature of spacetime at the micro scale.
iii). Interestingly the complexification of coordinates can also be related to

an underpinning of a Nelsonian-stochastic process [70].
3. Interestingly we can purse the reasoning of section 4, equations (57) ff to the case of terrestrial phenomena. Let us consider a gas at standard temperature and pressure. In this case, the number of molecules $n \sim 10^{23}$ per cubic centimeter, so that $r \sim 1cm$ and with the same l, we can get a "scaled" Planck constant $\tilde{h} \sim 10^{-44} << h$, the Planck constant.
In this case, a simple application of the WKB approximation, leads immediately from the Schrodinger equation at the new scale to the classical Hamilton-Jacobi theory, that is to classical mechanics.

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APPENDIX

As mentioned on using the double Weiner process and Newtonian mechanics, it is possible to derive the Schrodinger equation - this is the derivation of Quantum Mechanics from the Nelsonian stochastic theory. Let us briefly review the steps in this derivation. We first start with the backward and forward time derivatives in the double Weiner process,

$$\partial\rho/\partial t + div(\rho b_+) = \nu\Delta\rho,$$

$$\partial\rho/\partial t + div(\rho b_-) = -\nu\Delta\rho \tag{84}$$

Next we use the fact that if $\rho(\vec{r},t)$ is the probability density at $\vec{r}(t)$, then as demonstrated by Kolmogorov, for the above Weiner process and more generally any Markov process, we have forward and backward Fokker-Planck equations

$$\frac{d_+}{dt}x(t) = b_+ \quad , \quad \frac{d_-}{dt}x(t) = b_- \tag{85}$$

where, in the Nelsonian theory $\nu = \hbar/2m, \hbar$ being the reduced Planck constant. It is now possible from (2) to define the velocities

$$V = \frac{b_+ + b_-}{2} \quad ; \quad U = \frac{b_+ - b_-}{2} \tag{86}$$

Adding both the equations of (84) we get on using (86) the equation of continuity,

$$\partial \rho / \partial t + div(\rho V) = 0 \tag{87}$$

while on subtracting the two equations of (84) we get

$$U = \nu \nabla ln \rho \tag{88}$$

In the usual approach we define the complex velocity

$$\vee = V - \imath U$$

and on using a suitable Lagrangian derived from Newtonian Mechanics we deduce the Schrodinger equation for details). In any case it is to be noted that if the velocity U as given in (86) or (88) vanishes, that is the backward and forward time derivatives become equal, there is no double Weiner process and we have the usual Classical Theory, with $\nu = 0$. So Quantum Mechanics is contained in U).

Let us now not take recourse to Newtonian Mechanics, but merely define a function S such that

$$V = \nu \vec{\nabla} S \tag{89}$$

Using equations (88) and (89) it is possible to define a complex velocity potential ψ given by

$$\psi = \sqrt{\rho} e^{(\imath/\hbar)S} \tag{90}$$

We get the same ψ in the usual stochastic theory (as also in the hydrodynamical formulation), as can be easily verified. We now observe that substitution of V in terms of ψ from (90) in the equation of continuity (87) immediately leads to

$$\imath \hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi = 0 \tag{91}$$

(91) can be seen to be the free particle Schrodinger equation. We have not however used Newtonian Mechanics in the derivation of (91). In the usual Nelsonian Theory also, (91) is deduced, though via a Lagrangian.

If we now specialise to energy momentum eigen states, in the Quantum Mechanical Schrodinger equation (91), we obtain

$$E = p^2/2m \tag{92}$$

which expresses Newtonian Mechanics, as obtained from stochastic considerations. (For example a time derivative of both sides of (92) and a little algebraic manipulation, leads to Newton's first and second laws.)